On Modified and Reverse Wiener Indices of Trees

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The Wiener index is a well-known measure of graph or network structures with similarly useful variants of modified and reverse Wiener indices. The Wiener index of a tree T obeys the relation $W(T) = \sum_e n_{T,1}(e) \cdot n_{T,2}(e)$, where $n_{T,1}(e)$ and $n_{T,2}(e)$ are the number of vertices of T lying on the two sides of the edge e, and where the summation goes over all edges of T. The λ-modified Wiener index is defined as ${}^{m}W_{\lambda}(T) = \sum_e [n_{T,1}(e) \cdot n_{T,2}(e)]^{\lambda}$. For each $\lambda > 0$ and each integer d with $3 \leq d \leq n-2$, we determine the trees with minimal λ-modified Wiener indices in the class of trees with n vertices and diameter d. The reverse Wiener index of a tree T with n vertices is defined as $\Lambda(T) = \frac{1}{2}n(n-1)d(T) - W(T)$, where $d(T)$ is the diameter of T. We prove that the reverse Wiener index satisfies the basic requirement for being a branching index.

Key words: Modified Wiener Index; Reverse Wiener Index; Tree; Diameter.